

What Did They Mean by “Calculation Principles”?: Revisiting Argumentative Styles in Late Ming to Mid- Qing Chinese Mathematics*

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Introduction

Euclid's *Elements*, compiled at around 300 BCE in the Hellenistic city of Alexandria in Ptolemaic Egypt, is one of the most important pre-modern texts in the history of science and mathematics. The abstract, axiomatic, and deductive nature of the text was to become the paradigm for mathematics in Europe and in some other cultures for two millennia.¹ A good part of the historical context of the common term “proof” lies in Euclidean geometry, which was a central part of Western education.² It is impossible to overemphasize the importance of Euclid's *Elements* in the history of science and mathematics, especially concerning the development and the spread of the method of deduction.³

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¹ For the contents of Euclid's *Elements*, the reader may refer to Euclid, *The Thirteen Books of the Elements*, 3 vols. ed., Sir Thomas L. Heath (New York: Dover, 1956); for its influence, refer to, for instance, Morris Kline, *Mathematics in Western Culture* (New York: Oxford University Press, 1953).

² Judith V. Grabiner, “The Centrality of Mathematics in the History of Western Thought,” *Mathematics Magazine* 61:4 (1988), 220-230.

³ In this article, we use the terms “deduction/deductive reasoning” and “induction/inductive reasoning,” in the sense that is generally accepted by mathematicians and historians of mathematics. Roughly speaking, deduction is the kind of reasoning in which the truth of the premises guarantees the truth of the conclusion, provided that no mistake has been made in the reasoning. Deduction contrasts with induction, the kind of reasoning in which the truth of the premises need not

Euclid's *Elements* and its axiomatic-deductive system, i.e., starting from axioms that are assumed to be true and then proving propositions deductively with axioms or previously proven propositions, became a paradigm of Western thought. However, as twentieth-century historical research has already shown, there were very different styles of argumentation found in pre-modern mathematics in East Asia before its contact with Western science. For example, in Liu Hui's 劉徽 (fl. c. 263 CE) commentary on the classical Chinese mathematical text *Jiuzhang suanshu* 九章算術 (Nine Chapters on Mathematical Procedures, 1st century CE), he used an intuitive but convincing method involving limits and infinitesimals to demonstrate the volume formulae for several kinds of pyramids, which is very different from the logically-sound but counter-intuitive approaches used in Euclid's *Elements* on similar volume propositions.⁴ Moreover, classical and medieval Chinese mathematics was also transmitted to neighboring cultures, and teaching systems revolving around several Chinese classics and involving examinations existed in Korea and Japan.⁵ Therefore, we can safely assume that similar methods of argumentation were used not only in China but also in other parts of East Asia.

Although there existed very different argumentative styles in East Asia from those in Europe, most classical and medieval Chinese mathematical texts, along with the mathematical knowledge attached to them—including, for example, Liu Hui's commentary on the *Jiuzhang suanshu*—were lost in Ming China by the fifteenth century.

guarantee the truth of the conclusion. However, in a good inductive argument, usually with a number of examples, the premises provide some degree of support for the conclusion, where such support means that the truth of the premises indicates that the conclusion is likely to be true. The reader may refer to, for example, Joshua Schechter, "Deductive Reasoning," in Hal Pashler ed., *The Encyclopedia of the Mind*, <http://philpapers.org/rec/SCHDR> (accessed on July 3, 2016). Also refer to James Hawthorne, "Inductive Logic," in Edward N. Zalta ed., *The Stanford Encyclopedia of Philosophy* (Winter 2014 Edition), <http://plato.stanford.edu/archives/win2014/entries/logic-inductive> (accessed on July 3, 2016).

⁴ For general contents of the *Jiuzhang suanshu*, refer to Karine Chemla and Guo Shuchun, *Les neuf chapitres: Le classique mathématique de la Chine ancienne et ses commentaires* (Paris: Dunod, 2004); Liu Hui's arguments for the volumes of different pyramids is best described in Donald B. Wagner, "An Early Chinese Derivation of the Volume of a Pyramid," *Historia Mathematica* 6 (1979), 164-188; Euclid's arguments about the volume of a pyramid involve *Reductio ad absurdum*, and can be found in, for instance, Heath, *The Thirteen Books of the Elements*, Vol. 3, 378-395.

⁵ Jean-Claude Martzloff, *A History of Chinese Mathematics* (Berlin: Springer, 2006), 105-106.

Many scholars have claimed that during the Ming period traditional Chinese mathematics was in a state of decline.⁶ However, other scholars have also argued that in the Ming period the development of Chinese mathematics was not as stagnant as what has been described. With the emergence of a group of “Confucian-businessmen” in Ming society in the mid-fifteenth to the late sixteenth century, it seems there was another kind of popularized and commercialized mathematics that developed in China, focusing more on the mathematics of Chinese abacus and business.⁷

After this change in direction of mathematical developments, many of the lost mathematical texts were later recovered in the late eighteenth or early nineteenth century in the middle of the Qing Dynasty.⁸ During the period between loss and recovery, an important event occurred: European Jesuits went to China and started their missions in the late sixteenth century, beginning the transmission of Western science into East Asia. Therefore, a few interesting questions can be asked in this context. In general, what sorts of influence from European mathematics based on the paradigm set by Euclid’s *Elements* did Chinese scholars receive during the period when classical and medieval Chinese mathematics was generally lost? In particular, in the works by late Ming to mid-Qing Chinese mathematicians, the terms “principles” (*li* 理) or “calculation principles” (*suanli* 算理) are often seen in their mathematical arguments. What did they really mean by these terms? What were their argumentative styles like? What sorts of influence did European mathematics have on their works?

Many earlier scholarly works have tried to answer these questions. In particular, some of the studies related to the topic of this paper have stressed the positive and long-term influences of the translation of Euclid’s *Elements* in China. They claim that the translation and introduction of Euclid’s *Elements* brought deductive thinking into China and greatly helped the development of Chinese mathematics, and they

⁶ For the loss of traditional Chinese mathematics in the Ming period, refer to, for instance, Mei Rongzhao 梅榮照, “Ming-Qing shuxue gailun 明清數學概論 (An Introduction to the Mathematics of the Ming and Qing Periods),” in Mei Rongzhao, Wang Yusheng 王渝生, and Liu Dun 劉鈍, *Ming-Qing shuxueshi lunwenji* 明清數學史論文集 (A Collection of Papers about History of Mathematics in Ming and Qing Dynasties) (Nanjing: Jiansu jiaoyu chubanshe, 1990), 1-20.

⁷ Horng Wann-Sheng 洪萬生, *Mingdai shuxue yu shehui: 1368-1607 明代數學與社會: 1368-1607* (Mathematics and Society of Ming Dynasty), in Chu Pingyi 祝平一 ed., *Zhongguo shi xinxun: Keji yu zhongguo fence* 中國史新論: 科技與中國分冊 (A New History of China: Section of Science and Technology in China) (Taipei: Lianjing chubanshe, 2010), 353-421.

⁸ On the loss and recovery of classical and medieval Chinese mathematical texts in late imperial China, refer to, for instance, Li Yan and Du Shiran, *Chinese Mathematics: A Concise History*, tr., J. N. Crossley and A. W. Lun (Oxford: Clarendon Press, 1987), 226-230.

seem to have implicitly played down the influence of other texts of European origin.⁹ While this paper does not entirely object to their claims, we will try to present some Chinese reactions to the influence, and also discuss the impact made by other texts related to Western mathematics.

To achieve this goal, we have to see Chinese mathematicians from the late sixteenth century to the eighteenth century as more active practitioners of mathematics, and their efforts in digesting and re-inventing Western styles of argumentation should be understood in the context of the “integration” (*huitong* 會通) of Western and Chinese studies. Ever since the beginning of direct contact between East Asian and European civilizations, many Chinese scholars, faced with the challenges of completely different systems used to understand the human world and the universe, tried hard to thoroughly understand Western learning (especially concerning science and mathematics) in the hopes of assimilating or integrating it with Chinese learning to form a unified system. This was what the late Ming and early Qing scholars meant by *huitong*.¹⁰

Among the efforts of *huitong*, there are two important works that had a substantial impact on the styles of mathematical argumentation used by Chinese scholars. We shall first briefly describe the stories related to the two works, both of which have the same Chinese title, the *Jihe yuanben* 幾何原本.

Two Versions of the *Elements* from Europe

In 1607, the first six books of Christopher Clavius’ edition of Euclid’s *Elements*, translated by the Italian Jesuit Matteo Ricci (1552-1610) and the Chinese scholar Xu

⁹ For instance, the reader may refer to the following works: Mei, et al., eds., *Ming-Qing shuxueshi lunwenji* (cit. n. 6); Wang Yusheng, “Jihe yuanben tiyao 幾何原本提要 (An Abstract of the *Elements*),” in Guo Shuchun 郭書春 ed., *Zhongguo kexue jishu dianji tonghui: Shuxue pian* 中國科學技術典籍通匯：數學篇 (Sources Materials of Ancient Chinese Science and Technology: Mathematics Section) (Zhengzhou: Henan Education Press, 1993), Vol. 5, 1145-1150; Mei Rongzhao, Wang Yusheng, and Liu Dun, “Oujilide yuanben de chuanru he dui woguo ming qing shuxue de yingxiang 歐幾里德原本的傳入和對我國明清數學的影響 (The transmission of Euclid’s *Elements* and Its Influence on the Mathematics of the Ming-Qing Period of Our Country),” in Mei et al., eds., *Ming-Qing shuxueshi lunwenji* (cit. n. 6), 53-83.

¹⁰ For the efforts of *huitong* by Chinese scholars, refer to, for instance, Hashimoto Keizo and Catherine Jami, “From the *Elements* to Calendar Reform: Xu Guangqi’s Shaping of Mathematics and Astronomy,” in Catherine Jami, Peter Engelfriet, and Gregory Blue eds., *Statecraft and Intellectual Renewal in Late Ming China: The Cross-Cultural Synthesis of Xu Guangqi (1562-1633)* (Leiden: Brill, 2001), 263-278.

Guangqi 徐光啟 (1562-1633) with the title *Jihe yuanben*, were published in Beijing.¹¹ This is the first European mathematical text that was translated into Chinese.¹²

In their preface to the translation, Ricci and Xu dichotomize what they call mathematics into two categories of number (*shu* 數) and magnitude (*du* 度)¹³:

The school called “mathematics” specializes in observing the amounts and limits of things. The way to separate it is: if one cuts things into numbers, then it shows how many/much those things are; if one completes things with its magnitudes, then it shows how large those things are. For those numbers or magnitudes, one might detach them from objects and discuss them generally, and then the numbers set up the school of calculation methods, and the magnitudes set up the school of measurements.¹⁴

The terms *shu* and *du* are constantly used in earlier Chinese mathematical and astronomical texts. It would seem that Ricci and Xu tried to connect the newly introduced text with traditional Chinese learning, thus providing a legitimate basis for their *Jihe*

¹¹ Peter Engelfriet, *Euclid in China: The Genesis of the First Chinese Translation of Euclid's Elements in 1607 and Its Reception up to 1723* (Leiden: Brill, 1998). The second half of the title, *yuanben*, can be understood as “elements” or “origins,” while the first half is known to be misinterpreted as “geometry,” for the term “*jihe*” sounds like the prefix “geo.” Ricci and Xu seemed to use this term to include knowledge about not only geometry, but also numbers. See Mei, et al., eds., *Ming-Qing shuxueshi lunwenji* (cit. n. 6). The term “*jihe*” does indeed mean “geometry” in modern Mandarin, but this is also a term found in classical Chinese that means “how many/much.” In traditional Chinese mathematical writings, problems are usually posed with this term “*jihe*,” and this practice can be traced back to at least the beginning of early imperial China in the third and second centuries BCE. For instance, in the *Suan shu shu* 算數書 (Writings on Reckoning), which is dated no later than 186 BCE, the term “*jihe*” is used multiple times to mean “how many/much.” For *Suan shu shu*, refer to Christopher Cullen, “The *Suan shu shu* ‘Writings on Reckoning’: A Translation of a Chinese Mathematical Collection of the Second Century BC, with Explanatory Commentary, and an Edition of the Chinese Text” (Cambridge: Needham Research Institute Working Papers, 2004).

¹² Mei, et al., eds., *Ming-Qing shuxueshi lunwenji* (cit. n. 6), 7.

¹³ The translation of *shu* and *du* follows Engelfriet, *Euclid in China*, 139.

¹⁴ Guo ed., *Zhongguo kexue jishu dianji tonghui*, Vol. 5, 1151: “幾何家者，專察物之分限者也。其分者：若截以為數，則顯物幾何眾也；若完以為度，則指物幾何大也。其數與度，或脫於物體而空論之，則數者立算法家，度者立量法家也。” We translate the character “空” as “generally.” It is usually understood as “empty,” but in this context, since the discussions are supposed to be detached from concrete objects, “generally” fits the context. Xu seemed to be able to appreciate the “general” nature of Euclid’s propositions, although he did not compose his own texts in the same manner (see the next section).

yuanben. This kind of argument could have opened some space for the learning of Euclid. But it might also have put the newly introduced *Elements* mainly in the realm of real-world applications instead of an active field of research.¹⁵

Ricci and Xu's strategy worked. This 1607 version seemed to have circulated widely among those Chinese scholars in the seventeenth century who were interested in mathematics.¹⁶ Moreover, the dichotomy of number and magnitude became a new way for contemporaneous scholars to re-categorize and understand traditional Chinese mathematics, as evidenced by Mei Wending's 梅文鼎 (1633-1721) *Fangcheng lun* 方程論 (Arguments on the Method of Rectangular Arrays, 1672):

Mathematics is one complete learning. If separated, there are "magnitude" and "number." "Magnitude" is the methods of measurements, and "number" is the procedures of calculation. Both of them develop from the shallower to the deeper. Therefore, the methods of measurements have the easiest method of "field measurement," then the slightly advanced methods of "short width" and "construction consultations," and finally reach the limit of "base and height." The procedures of calculation have the easiest method of "millet and rice," then the slightly advanced methods of "proportional distribution," "equitable transport," and "excess and deficit," and finally reach the apex of "rectangular array."¹⁷

As the reader can see, the names of the methods under the two categories are titles of the chapters of the *Jiuzhang suanshu*, the most important mathematical canon and a paradigm for the learned practice of mathematics in pre-modern China. Thus Mei was using Ricci and Xu's framework to understand traditional Chinese mathematics, a clear effort of *huitong*.

Mei Wending also noticed the importance of mathematical arguments. Later in his *Fangcheng lun*, he explains the title of his work and uses the term *suanli*:

If a mathematical book has examples but no arguments, then [the reader] would not know the origin of the solutions. This is the reason for the multitude of mistakes when it is copied and circulated. This book wishes to clarify the *calculation principles*, so there are more arguments than examples.¹⁸ In the beginning of each volume, there is always a general argument as its guideline, and then I give examples to solidify the claims. ... Generally, seven tenths [of the contents] are

¹⁵ Engelfriet, *Euclid in China*, 452.

¹⁶ Mei, et al., eds., *Ming-Qing shuxueshi lunwenji* (cit. n. 6), 66.

¹⁷ Guo ed., *Zhongguo kexue jishu dianji tonghui*, Vol. 4, 324-325: "夫數學一也。分之有度有數，度者量法，數者算術。是兩者皆由淺入深。是故量法最淺者方田，稍進為少廣，為商功，而極於勾股。算術最淺者粟米，稍進為衰分，為均輸，為盈朒，而極於方程。"

¹⁸ The emphasis with italics is the authors'.

arguments, while three are examples. To name the book with the word “argument” is to highlight the contents.¹⁹

Moreover, a few folios after his explanation of *suanli*, Mei says that he wanted to confirm his findings with the “ancient text,” but he could not do it as the ancient text was not available.²⁰ This shows that he did not actually see the entirety of the *Jiu-zhang suanshu*. Mei Wending was creative and original in his mathematical studies, but the foundation of his studies was mathematics from Ming China and works from Europe, especially the translation of Euclid’s *Elements*.

In fact, if we take a look at how the original Chinese translator of the 1607 version talked about the text and its arguments, he gave extremely high praise. In Xu Guangqi’s own *Jihe yuanben zayi* 幾何原本雜議 (Miscellaneous Discussions on the *Elements of Mathematics*, early 17th century), there is this paragraph:

There are four non-necessities for this book: there is no need to doubt, no need to guess, no need to try, and no need to change. There are four impossibilities: it is impossible to skip some part of it, impossible to refute it, impossible to reduce it, and impossible to swap what comes earlier for what comes later. There are three extremes and three capabilities: it seems to be extremely obscure, but it is in fact extremely clear, so it is capable of clarifying the extreme obscurities of others; it seems to be extremely complex, but it is in fact extremely simple, so it is capable of simplifying the extreme complexities of others; it seems to be extremely difficult, but it is in fact extremely easy, so it is capable of making easy the extreme difficulties of others. Easiness comes from simplicity, and simplicity comes from clarity. To sum up its wonder, it lies in none other than the clarity.²¹

It would seem, at least from Xu’s own words, that he understood the importance and the usefulness of the axiomatic-deductive nature of Euclid’s *Elements*. We shall come back to this issue in the next section, but now we will introduce another text with the same Chinese title, *Jihe yuanben*, published in 1723.

There is another version of the *Elements* written in Chinese, also edited according

¹⁹ Guo ed., *Zhongguo kexue jishu dianji tonghui*, Vol. 4, 325: “算學書有例無論，則不知作法根源。一再傳而多誤，蓋由於此。本書欲明算理，故論多於例。每卷之首，皆有總論以為題綱，然後舉實例以實其說。…凡為論者十之七，而例居其三。以論名篇，著其實也。”

²⁰ Guo ed., *Zhongguo kexue jishu dianji tonghui*, Vol. 4, 326.

²¹ Guo ed., *Zhongguo kexue jishu dianji tonghui*, Vol. 5, 1158: “此書有四不必：不必疑，不必揣，不必試，不必改。有四不可得：欲脫之不可得，欲駁之不可得，欲減之不可得，欲前後更置之不可得。有三至三能：似至晦，實至明，故能以其明明他物之至晦；似至繁，實至簡，故能以其簡簡他物之至繁；似至難，實至易，故能以其易易他物之至難。易生於簡，簡生於明。綜其妙，在明而已。”

to a European mathematical text. This text is again titled *Jihe yuanben*.²² This second *Elements* was written as a part of Emperor Kangxi's 康熙 (r. 1662-1722) policy to appropriate Western science for his empire. Kangxi's interest in Western knowledge was constant throughout his reign, after he first expressed concerns about the calendar in 1668. Although he had officially assumed personal rule a year earlier, the regent Oboi was still the *de facto* ruler of the empire. At the time, a severe dispute was raging between some Chinese scholars and Jesuit astronomers in his court. Kangxi's settling of the dispute to the latter's advantage was actually a decisive step in his assertion of power against Oboi. Kangxi later expressed that, after looking closely into the matter, he felt indignant at his ministers' ignorance of astronomy and ashamed of his own.²³

Kangxi subsequently studied astronomy and mathematics from the Jesuits, and he also had his sons schooled in them, as part of their broader training.²⁴ One set of the lecture notes for the emperor, with the Chinese title as none other than "*Jihe yuanben*," was written by the French Jesuits Jean-François Gerbillon (1654-1707) and Joachim Bouvet (1656-1730) when they served as Kangxi's court savants in the 1690s. One of Kangxi's purposes in studying geometry was to understand practical geometry, and the lecture notes reflect that purpose by adding a chapter of practical geometry with topics such as measurement and map-making, which were not traditionally considered by Europeans as Euclidean geometry.²⁵ The Jesuits' lecture notes were based on a French geometry textbook which was popular in Europe, the *Elémens de géométrie* by the Jesuit Ignace-Gaston Pardies (1636-1673).²⁶

²² As mentioned above, the term "*jihe*" could be used by Ricci and Xu to mean "mathematics." But in this case, since the contents in the text were entirely about geometry, it would seem that when the text was being written by the Jesuits missionaries (see below), they were using the term to mean "geometry." So the title of this particular text can be understood as the "Elements of Geometry." See Tian Miao 田淼, *Zhongguo shuxue de xihua licheng* 中國數學的西化歷程 (The Westernization of Mathematics in China) (Jinan: Shandong Education Press, 2003), 26-38.

²³ Catherine Jami, "'European Science in China' or 'Western Learning'?: Representations of Cross-Cultural Transmission, 1600-1800," *Science in Context* 12:3 (1999), 413-434. The calendar dispute, its political aftermath, and later influences are also discussed in detail in Huang Yi-Long, "Court Divination and Christianity in the K'ang-hsi Era," *Chinese Science* 10 (1991), 1-20. Or refer to Chu Pingyi, "Scientific Dispute in the Imperial Court: The 1664 Calendar Case," *Chinese Science* 14 (1997), 7-34.

²⁴ Jami, "'European Science in China' or 'Western Learning'?", 427.

²⁵ Catherine Jami, *The Emperor's New Mathematics: Western Learning and Imperial Authority during the Kangxi Reign (1662-1722)* (New York: Oxford University Press, 2012), 173-176.

²⁶ Catherine Jami, "From Clavius to Pardies: The Geometry Transmitted to China by Jesuits (1607-1723)," in F. Masini ed., *Western Humanistic Culture Presented to China by Jesuit Missionaries (XVII-XVIII centuries)* (Rome: Institutum Historicum S. I., 1996), 175-199.

Although this second *Elements* was originally a set of lecture notes, it was later included in a very influential compendium. In 1723, an imperial canon of mathematics was published with the title *Yuzhi Shuli jingyun* 御製數理精蘊 (Essential Principles of Mathematics, Imperially Composed; from here on abbreviated as the *Shuli jingyun*), in which most of the contents of the Jesuits' lecture notes are included in its second to fourth volumes, again named the *Jihe yuanben*.²⁷ Since this text was regarded as a canon by the empire, it circulated widely and became very influential from its publication in the early eighteenth century. The imperial canon was first used in the Banner Schools, and then later in the Imperial University, so the text became available to the Chinese in general as well as to bannermen. The text continued to be used until the end of the nineteenth century.²⁸ The wide circulation and broad application of the imperial canon certainly had an influence on Chinese scholars. In the next two sections, we shall discuss the influence these two versions of the *Elements* had in terms of argumentative styles used by Chinese scholars in different periods.

Argumentative Styles in the Late Ming and Early Qing Periods

The wave of the introduction of Western learning to East Asia from the late sixteenth to the eighteenth century started a trend for Chinese scholars to study European mathematics, from different purposes ranging from intellectual curiosity to calendrical reform. Some Chinese scholars tried to integrate Western mathematics with the Chinese counterpart that was available to them, and we can see this from the argumentative styles in the works from this period.

As mentioned above, Xu Guangqi, as one of the translators of the 1607 version of Euclid's *Elements*, was apparently well versed in the axiomatic-deductive system. It is possible to see to what extent Xu was actually influenced by this manner of argumentation, and how he integrated his Chinese mathematics training with its European counterpart. After the 1607 version of Euclid's *Elements*, he helped translate, again with Matteo Ricci, a text about measurement, the *Celiang fayi* 測量法義 (Methods and Meaning of Measurement, 1608). In the preface to the *Celiang fayi*, there is severe criticism of earlier Chinese mathematicians. Xu seemed to believe that earlier Chinese mathematicians, from the legendary sages to several famous scholars, could not explain even the simple “meaning” of the Pythagorean Theorem and that of some measurement methods, and one could not understand them without

²⁷ Han Qi 韓琦, “Shuli jingyun tiyao 數理精蘊提要 (Abstracts of the *Shuli jingyun*),” in Guo ed., *Zhongguo kexue jishu dianji tonghui*, Vol. 3, 1-10.

²⁸ In 1882, a revised version was published under the supervision of some officials of the Ministry of War. See, for instance, Jami, *Emperor's New Mathematics*, 382-384.

studying Euclid’s *Elements*.²⁹

Xu Guangqi later wrote the *Celiang yitong* 測量異同 (Differences and Similarities [between Traditional and Western Methods] of Measurement, 1608) and the *Gougu yi* 勾股義 (Meaning of the Methods of the Right-Angled Triangle, 1609). The subject of *gougu* 勾股, or the theories of right-angled triangles, is a good place to see the Chinese efforts for *huitong*, for most of the traditional geometry except this topic in the *Jiuzhang suanshu* was lost in that period, and there was a trend during the Ming Dynasty to study this subject extensively.

Xu Guangqi’s interest in *gougu* might be due to its real-world applications.³⁰ We can see his approaches in integrating Chinese and European mathematics in the *Gougu yi*. In this work, many of the famous problems about right-angled triangles in traditional Chinese works are re-written, and the “meaning” behind the calculation procedures is explained by Xu. We use problem 14 in the text to show Xu’s argumentative style. The problem, if we use modern notation and represent the two legs and hypotenuse of a right-angled triangle as a , b , and c , is about finding the three sides of a right-angled triangle given $(c - a)$ and $(c - b)$. This problem is in fact included in essentially every mathematical text from the Ming Dynasty that includes some discussion about right-angled triangles.³¹ After the introduction of Western mathematics, the vertices of geometrical diagrams in Chinese mathematical texts are usually labelled with Chinese characters, just as those in European texts are labelled with Greek or Latin letters. These kinds of diagrams are called “labelled diagrams” in this paper. Chinese scholars often used 10 “heavenly stems” (*tiangan* 天干) and 12 “earthly branches” (*dizhi* 地支) to label vertices in a diagram. For the sake of convenience, we use Latin letters to transcribe them.³² What follows is Xu

²⁹ For Xu’s preface, see Guo ed., *Zhongguo kexue jishu dianji tonghui*, Vol. 4. p. 5. Xu Guangqi mentioned Liu Hui’s name in this preface. In fact, Liu Hui did give very convincing proofs for many procedures in the *Jiuzhang suanshu*, but since the ancient classic and Liu’s commentaries were not available in Xu’s time, it is understandable that Xu would think Liu’s arguments were similar to many in the Ming mathematical works, which, according to the standards of Euclid, could not explain the “meaning” of the procedure very well.

³⁰ See the preface of the *Celiang fayi* and the *Gougu yi* in Guo ed., *Zhongguo kexue jishu dianji tonghui*, Vol. 4, pp. 5, 28.

³¹ See, for instance, Guo ed., *Zhongguo kexue jishu dianji tonghui*, Vol. 2, pp. 273, 748, 984, 1371.

³² The following table shows the correspondences between Chinese label characters and the Latin letters we use.

<i>tiangan</i>	甲	乙	丙	丁	戊	己	庚	辛	壬	癸		
Latin letters	A	B	C	D	E	F	G	H	I	J		
<i>dizhi</i>	子	丑	寅	卯	辰	巳	午	未	申	酉	戌	亥
Latin letters	K	L	M	N	O	P	Q	R	S	T	U	V

Guangqi’s wording of this problem and its solutions:

[Given] the difference between the height and the hypotenuse, and that between the base and the hypotenuse, find the base, the height, and the hypotenuse.³³

The method says: The height AB and the hypotenuse AC have difference 2; the base BC and the hypotenuse AC have difference 9; find the base, the height, and the hypotenuse. Multiply the two differences and one obtains 18. Double it and one obtains 36 as the dividend. Extract its square root and one obtains 6, which is the difference between the sum [of the two legs] and the hypotenuse. Add it to 9, the difference between the base and the hypotenuse, one obtains 15, the height AB. Add it to 2, the difference between the height and the hypotenuse, one obtains 8, the base BC. Add the base to the difference between the base and the hypotenuse, or add the height to the difference between the height and hypotenuse, one obtains 17, the hypotenuse AC.³⁴

Even before reading into his arguments for the correctness of the proposed procedures, several points can already be seen from this excerpt of Xu’s work. Firstly, the opening sentence is the general description of the problem, while the second sentence is the problem rephrased in terms of what is given and what is sought, with clear reference to certain parts in a diagram. These first two sentences are obviously modelled from the first two divisions of the format of a proposition in the *Elements*: the “enunciation” and the “setting-out.”³⁵

Secondly, the solution refers to a diagram (see Figure 1) in which vertices are labelled. In traditional Chinese mathematical texts, the labelled components in a diagram are usually the sides, blocks, or faces, but not the vertices. This practice of labelled vertices again follows the Euclidean tradition.³⁶

Thirdly, the reader should have already noticed that although the “enunciation”

³³ Here we translate the two legs of the right-angled triangle a and b as “base” and “height,” for the reason that in traditional Chinese mathematics, a right-angled triangle is usually oriented in the way that the shorter of the two legs is drawn horizontally (thus the “base”) and the longer of the two legs vertically (thus the “height”).

³⁴ Guo ed., *Zhongguo kexue jishu dianji tonghui*, Vol. 4, 37: “股弦較、勾弦較，求勾、求股、求弦。法曰：甲乙股，甲丙弦，較二。乙丙勾，甲丙弦，較九。求勾、求股、求弦。以二較相乘得十八，倍之，得三十六，為實。平方開之，得六，為弦和較。加勾弦較九，得甲乙股十五。加股弦較二，得乙丙勾八。以勾弦較加勾，或股弦較加股，得十七為甲丙弦。”

³⁵ Heath ed. and tr., *Elements* (cit. n. 1), 129.

³⁶ For an extensive discussion about Greek diagrams, refer to Reviel Netz, *The Shaping of Deduction in Greek Mathematics: A Study in Cognitive History* (Cambridge: Cambridge University Press, 1999).

ing's *Gougu juyu* 勾股舉隅 (Giving Examples about the Methods of the Right-Angled Triangle, late 17th or early 18th century).³⁷

Mei Wending's own studies on the subject of *Gougu* also show his efforts in integrating Chinese and European mathematics. The following is a part of the demonstration and its diagram of the same problem in the previous example written by Xu Guangqi (finding the three sides of a right-angled triangle given $[c - a]$ and $[c - b]$) (see Figure 3).

The argument says: AB is the square for the self-multiplied hypotenuse, and AD is the square for the self-multiplied height. Mutually subtract the two squares, and the remaining bent shape CIHBGKF is equal to EB, the square for the self-multiplied base. But CH is the oblong of the difference between height and hypotenuse (CI) multiplied by the difference between the base and the hypotenuse (IH), which is equal to FG.³⁸ These two oblongs [combined] must be equal to the square ED. The square ED is the self-multiplication of the difference between the sum [of two legs] and the hypotenuse (The base EG subtracts JG, the difference between the height and the hypotenuse, and the remainder JE is the difference between the sum and the hypotenuse). Therefore, [if one] doubles the product of the two differences between the base and the hypotenuse, and between the height and the hypotenuse, and then extracts the square root, then one obtains the difference between the sum [of two legs] and the hypotenuse.³⁹

It can be seen that Mei uses a labelled diagram (a trait of European mathematics) and an intuitive manipulation of components in the diagram (a trait of Chinese math-

³⁷ Du Zhigeng was a scholar living in the late seventeenth century. He received the degree of *Juren* 舉人 from the Imperial Examination in 1687. Du studied Western mathematics when he was young, and two mathematical works of his, the *Shuxue yao* and the *Jihe lunyue* 幾何論約 (Simplified Discussions of Geometry, 1700), survive to this day. His purpose for writing mathematical works seems to have been to popularize mathematics. For Du Zhigeng's life and works, the reader may refer to, for instance, Cheng Heqin 程和欽, "Qingdai suanxuejia Du Zhigeng ji qi *Shuxue yao* zhi yanjiu 清代算學家杜知耕及其《數學綱》之研究 (A Study on the Qing Dynasty Mathematician Du Zhigeng and His *Key to Mathematics*)" (Master diss., National Taiwan Normal University, 2006). The work by Mei Wending was published posthumously in 1759 by his grandson. Refer to Guo ed., *Zhongguo kexue jishu dianji tonghui*, Vol. 4, 431-432.

³⁸ Words in parentheses refer to the author's self-commentaries written in the smaller characters printed in the source. In the quotation we also put those self-commentaries in smaller characters.

³⁹ Guo ed., *Zhongguo kexue jishu dianji tonghui*, Vol. 4, 439: "論曰：甲乙為弦自乘方，甲丁為股自乘之方。兩方相減，餘丙壬辛乙庚子己磬折形，與勾自乘戊乙方等。而丙辛為股弦較丙壬乘勾弦較壬辛之長方，與己庚等。此兩長方必與戊丁正方等，戊丁方者，弦和較自乘也。戊庚勾內減去癸庚股弦較，餘戊癸為弦和較。故倍勾弦股弦兩較相乘為實，開方而得弦和較也。"

ematics) to argue for the problem. Also, this problem and Mei’s demonstrations certainly look like he tries to respond to Xu’s argument by writing a more general, and more “Euclidean,” argument, because Mei not only uses no numerical example, but also draws a diagram with only labelled vertices instead of blocks (see Figure 3).

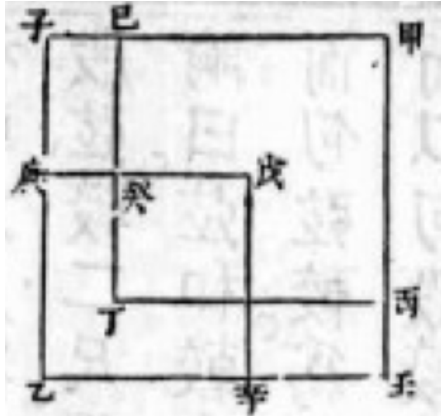


Figure 3. Diagram for the argument of a famous problem in the *Gougu juyu*. From Guo ed., *Zhongguo kexue jishu dianji tonghui: shuxue pian*, Vol. 4, 439.

On the other hand, the *Shuxue yao* constitutes Du Zhigeng’s own demonstrations for the solutions of different important problems in Chinese mathematics, around the same time as Mei Wending, and Du’s demonstrations also involve labelled diagrams and certain specific examples. The quotation in the next paragraph is from Rule 29 in Volume 1 of the text.

Find the perpendicular line from the opposite angle in a triangle of two equal sides.

Suppose a triangular field has the base with the width of 6 *bu*, and the two remaining sides are 5 *bu*; find the middle length. The method says: Set the base and halve it (one obtains 3 *bu*);⁴⁰ multiply it with itself (one obtains 9 *bu*); also multiply the remaining side with itself (one obtains 25 *bu*); subtract one number from the other (the remainder is 16 *bu*); extract the square root and one obtains 4 *bu*, which is what is sought.

⁴⁰ As in the previous problem, sentences in parentheses refer to the author’s commentaries written in the smaller characters in the source.

Explanation says: Take CB as the hypotenuse, BD as the base, and the sought CD as the height. This is the method of finding the height, given the base and hypotenuse (Volume 6, Rule 2). Halve AB to obtain the base, and the reason is that the two sides BC and CA are equal.⁴¹

This is a fairly straightforward problem of finding the height given the base and two remaining sides of an isosceles triangle. As the reader can see, a general method is again argued with a numerical example. The difference in Du's writing from Xu Guangqi's is that at the end of the paragraph, he tries to add a few lines of more general reasoning without referring to any specific number.

The argumentative styles of Xu Guangqi, Mei Wending, and Du Zhigeng shown in this section are by no means unique examples. They are a common practice of Chinese mathematics in the late Ming and early Qing periods. Labelled diagrams are probably the first Euclidean trait one might notice in their geometrical arguments. But other than this, they were still inclined to use numerical examples to demonstrate the correctness of a method, although they did sometimes use some general, or more abstract, descriptions in the process of their arguments, as seen in Mei Wending's case. All three of them show different levels of *huitong*, or integration, of European and Chinese mathematics. This practice of combining European and traditional Chinese argumentative styles can be called "Sino-Western mathematics."

The culmination of this practice of Sino-Western mathematics is the compilation and publication of the aforementioned imperial mathematical canon, the *Shuli jingyun*. The intuitive nature of the geometrical arguments in Pardies' *Eléments de géométrie* was well used by the compilers of the imperial canon.⁴² For instance, they used both general descriptions and numerical examples, just as late Ming and early

⁴¹ Du Zhigeng, *Shuxue yao* 數學鑰 (1681), Vol. 1, 18b-19a. In *Siku quanshu* 四庫全書 (Complete Library in Four Sections), *Zibu* 子部 (Section of Masters), Vol. 6, Category of Astronomy and Mathematics: "兩邊等之三角形求對角之垂線。設三角田底闊六步，兩餘邊各五步，求中長。法曰：置底折半得三步。自乘得九步。餘邊亦自乘得二十五步。兩數相減餘一十六步。平方開之得四步即所求。解曰：丙乙作弦，乙丁作勾，以所求之丙丁作股。此即勾弦求股法也。六卷二則。甲乙邊折半得勾者，以乙丙、丙甲兩邊等也。"

⁴² For the intuitive nature of the arguments in Pardies' work that was transmitted to East Asia, see, for instance, Jami, "From Clavius to Pardies" (cit. n. 26). In this paper, we use the terms "intuition/intuitive arguments" also in the sense that are used by mathematicians or educators. They mean a revelation or understanding about a proposition, obtained through mental manipulations about certain mathematical objects, or simple observation of some physical representations of those objects. An intuitive argument is not a "proof," but the person who achieves some intuition about a proposition usually has some sense about why a proposition is true. The reader may refer to L. Burton, "Why Is Intuition So Important to Mathematicians but Missing from Mathematics Education?," *For the Learning of Mathematics* 19:3 (1999), 27-32.

Qing scholars. When it comes to arguments related to methods of limit and infinitesimals, let us take a closer look at an example. What follows is a part of the paragraph in the imperial canon about the volume of a sphere.

For every sphere whose surface area is equal to the base area of a cone, and whose radius is equal to the height of the cone, the volume of the sphere is equal to the volume of the cone. ... Try to divide the sphere from the center into tens of thousands of pointed solids, and also divide the cone into tens of thousands of pointed solids. Then each pointed solid divided from the sphere is equal to each pointed solid divided from the cone. Why is it the case? The pointed solids divided from the sphere are based on the surface of the sphere and have the radius ... of the sphere as the height. The pointed solids divided from the cone are based on the base of the cone and have the height ... of the cone as the height. The base area of the cone is equal to the surface area of the sphere. And the height of the cone is equal to the radius ... of the sphere. So the two kinds of tens of thousands of pointed solids are of the same base and height, so there is no doubt that their volumes are equal. ... Since the divided solids are equal, it can be understood that the original solids must also be equal.⁴³

As the reader can see, when they needed to use arguments about infinitesimals, such as when demonstrating the volume formula of the sphere, they asked the reader to imagine there were “tens of thousands” of small “pointed solids” that were cut out from the sphere and reassembled. In another case, they even went further to ask the reader to create wooden or stone models to understand the volume formulae of pyramids and cylinders.⁴⁴ These examples show that the arguments do not appeal to the logical connections in a system, but to the reader’s intuition. Kangxi’s reason for studying geometry, both for himself and for his empire, was practical, so there seemed to be no need to spend too much time on the foundation and rigor of geometry.⁴⁵ Appealing to intuition is a possible way of quick learning and immediate applications if necessary. Therefore, it should not be difficult to understand the intuitive nature of the arguments in his imperial mathematical canon.

⁴³ Guo ed., *Zhongguo kexue jishu dianji tonghui*, Vol. 3, 60: “凡球體外面積，與尖圓體之底積等，而球體之半徑，與尖圓體之高度等，則此球體之積與尖圓體之積等也。... 試將球體從中心分為千萬尖體，復將尖圓體亦分為千萬尖體，則球體所分尖體每一分，必與尖圓體所分尖體等。何也？蓋球體所分尖體，皆以球體之外面為底，而以球體之 ... 半徑為高。其尖圓體所分尖體，皆以尖圓體之底為底，而以尖圓體之 ... 高為高。夫尖圓體之底積，原與球體之外面積等。而尖圓體之高度，又與球體 ... 半徑等。故此兩種千萬尖體，皆為同底同高，其積相等無疑矣。... 其所分之體既等，則原體亦必相等可知。”

⁴⁴ Guo ed., *Zhongguo kexue jishu dianji tonghui*, Vol. 3, 59.

⁴⁵ Jami, *Emperor’s New Mathematics*, 161.

The two versions of the *Elements* coexisted in the Qing Empire and some of its neighbors from the early to mid-eighteenth century. Although the original European versions have very different styles of composition and argumentation, they both seemed to have been consistently influential in China. In what follows we will present some examples of argumentative styles in the works of the scholars of the mid-Qing period.

Argumentative Styles in the Works of Scholars in the Early to Mid-Qing Period

As the reader has already seen from Mei Wending's words, "meaning" or "calculation principles" had been an important focus for Chinese scholars to study mathematics since the early Qing period. Through the reading of Ricci and Xu's translation, and the process of the compilation of the *Shuli jingyun*, Chinese mathematicians gradually used both deduction, though relatively intuitive, and induction, such as numerical examples, in their arguments.

In fact, the process of compiling the *Shuli jingyun* provided the opportunity for some Chinese, Manchu, and Mongolian mathematicians to reflect upon "calculation principles" and argumentative styles. The Mongolian astronomer Minggatu (late 17th to late 18th century) argued in 1774 for the correctness of the three "Jartoux's formulae" and further derived six identities on the basis of the former three.⁴⁶ Minggatu's work was criticized by Dong Youcheng 董祐誠 (1791-1823), who was later in turn criticized by Xiang Mingda 項名達 (1789-1850). In both cases the criticism was about not giving convincing enough arguments.⁴⁷ Whether their criticism was valid or not, it shows that scholars in the mid-Qing period were no longer satisfied with conclusions reached only with numerical examples or simple induction. They also cared about the principles behind algorithms and formulae. They seemed to have demanded more rigor in mathematical arguments. In what follows we shall give two examples from the mid-Qing scholars Li Huang 李潢 (?-1812) and Jiao Xun 焦循 (1763-1820) and discuss their argumentative styles.

The *Haidao suanjing xicao tushuo* 海島算經細草圖說 (Detailed Calculations and Graphical Explanations for the *Sea Island Mathematical Canon*, 1820) is Li Huang's re-explanation of ancient Chinese measurement problems recorded in the *Haidao*

⁴⁶ Catherine Jami, "Western Influence and Chinese Tradition in an Eighteenth-Century Chinese Mathematical Work," *Historia Mathematica* 15 (1988), 311-331.

⁴⁷ Guo ed., *Zhongguo kexue jishu dianji tonghui*, Vol. 5, 411, 473.

$$AB = [CD \cdot DG / (GH - DE)] + CD$$

What follows is Li Huang's demonstration of the procedure.

Construct FCO parallel to GEDB, then OB is equal to both CD and FG. Construct FI parallel to CE, then IG is equal to ED. The shape FGH is similar to the shape AOF, and the shape FGI is similar to the shape AOC. The shape FGH and its part, the shape FGI, have the same base FG and HI is their difference of heights. The shape AOF and its part, the shape AOC, have the same base AO and FC is their difference of heights. To compare the small height difference HI to the small base FG is like to compare the large height difference FC to the large base AO. Once you find AO, you can add the height of the measure OB (which is equal to CD), and then you obtain AB as the height of the island.⁵⁰

As the reader can see, Li Huang uses the concept of similarity (*tongshi* 同式) to argue for the procedure, but the similarity of geometrical shapes is not a Chinese concept but rather is taken from Euclidean geometry.⁵¹ Li Huang also uses a labelled diagram. After the demonstration of the general procedure, Li Huang then solves the problem numerically with the values given in the initial statements of the problem.

Li Huang uses a concept and a diagram from Western mathematics to give a general argument for the truth of a famous procedure in traditional Chinese mathematics, and then he solves the problem numerically in the text. This again shows the mathematician's efforts of *huitong*.

Aside from the topics that can be categorized as geometry, we can also find texts about other topics in mathematics that show the tendency of *huitong*, such as Jiao Xun's work. From the title of his *Jiajian chengchu shi* 加減乘除釋 (Explanations for Addition, Subtraction, Multiplication, and Division, 1794), one might assume it is a simple text about the four basic operations. However, if one takes a closer look at it, one finds that besides solving simple arithmetic problems, Jiao Xun seemed to have followed Mei Wending's idea of clarifying the calculation principles, not just for problems about geometry but also about arithmetic, and he set a goal of resting the foundation of all the principles on those of the four basic operations. This can be observed from the following paragraph:

⁵⁰ Guo ed., *Zhongguo kexue jishu dianji tonghui*, Vol. 4, 1200: “與庚戌丁乙平行作己丙辰，則辰乙與丙丁、己庚俱等。與丙戌平行作己壬，則壬庚與戊丁等。己庚辛形與甲辰己形同式，己庚壬形與甲辰丙形同式。己庚辛形分己庚壬形，同己庚句而辛壬為股較。甲辰己形分甲辰丙形，同甲辰句而己丙為股較。以小股較辛壬比小句己庚，若大股較己丙比大句甲辰。既得甲辰表高辰乙即丙丁得甲乙為島高。”

⁵¹ For the lack of the concepts “angle” and “similarity” in traditional Chinese mathematics, refer to, for example, Christopher Cullen, *Astronomy and Mathematics in Ancient China: The Zhou Bi Suan Jing* (Cambridge: Cambridge University Press, 1996), 77-80.

Westerner Sajilide’s book *Jihe yuanben* is particularly adept in speaking of shapes. Mei Wuan (Mei Wending) clarified it with the principles of *gougu*. When one discusses shapes, there is no chance that one does not base the arguments on *gougu*. And when discussing numbers, there is no chance that one does not base the arguments on addition, subtraction, multiplication, and division. Those who study [mathematics] know shapes from numbers, and use numbers from shapes, all due to the principles of addition, subtraction, multiplication, and division, so they can understand the excellence of [the principles behind] square, circle, area, and volume.⁵²

In fact, Jiao Xun also tried to re-examine many traditional mathematical texts (mainly the *Jiuzhang suanshu*) with the tool of “calculation principles” in his *Jiajian chengchu shi*.⁵³ He specifically mentioned the ancient classic in one of his letters to another mathematician:

The reasons for why the [procedures in] the *Jiuzhang suanshu* are the way they are, are discussed by people of earlier generations, but [the earlier generation] could not be clear [about the reasons]. I tried to understand its excellence of calculations, and the reason exists only in addition, subtraction, multiplication, and division.⁵⁴

For this reason, Jiao Xun wrote his *Jiajian chengchu shi*. In the text, he mainly used heavenly stems as symbols to discuss a total of 98 rules, not only about the four basic operations, but also various “principles of calculations” found in ancient classics. He explains in his work why those principles are correct, without using specific numerical examples, but sometimes with both abstract descriptions and diagrams. After showing the general principles, he uses those principles to solve numerical problems quoted from ancient classics.

Concerning his argumentative style, we give the following example. In the first half of the 11th rule in the text, the words are:

⁵² Guo ed., *Zhongguo kexue jishu dianji tonghui*, Vol. 4, 1340-1341: “西人薩几里得幾何原本一書，精於說形，梅勿菴明以句股之理。夫論形，未有不本諸句股。猶論數，未有不本諸加減乘除也。學者由數知形，由形以用數，悉諸加減乘除之理，自可識方圓纂積之妙。”

⁵³ Many of the lost texts about traditional Chinese mathematics were recovered in the late eighteenth and early nineteenth century. On the recovery of the *Jiuzhang suanshu* in China, refer to, for instance, Li and Du, *Chinese Mathematics* (cit. n. 8), 226-230.

⁵⁴ Horng Wann-Sheng 洪萬生, *Tantian sanyou* 談天三友 (Three Friends Talking about the Heaven) (Taipei: Mingwen shuju, 1993), 144: “九章所以然之故，前人頗言之，而未能了然。循嘗以推算之妙，其轉移變化，惟在加減乘除。”

Add A to B , and add B to A , then their differences are levelled; add A to A , and add B to B , then their differences are doubled.⁵⁵

In modern notations, the statement can be rendered as $(A + B) - (B + A) = 0$ and $(A + A) - (B + B) = 2(A - B)$. After the statement, the argument in the next paragraph runs as follows:

A is originally excessive, and one eliminates [the excess] with B . B is originally deficient, and one supplements it with A . Therefore, [what begins with] difference turns out to be no difference. ... A is originally excessive but one supplies it with A . B is originally deficient, but one only supplies it with B . there are two A and two B , so one doubles the difference between A and B .⁵⁶

Interestingly, Jiao Xun uses diagrams to help his arguments. He first lists four rectangles A , B , C , and D with different lengths. And then at the bottom the first diagram represents $A + B$ and $B + A$, with the same length on two sums and marked *chaping* 差平 (difference levelled). The second diagram represents $A + A$ and $B + B$, with the term *chabei* 差倍 (difference doubled) marked on the difference between two sums (see Figure 5).

As this example shows, Jiao Xun's argumentative style is to use a set of symbols (though different from Western symbolic algebra) to argue for the validity of general statements, without appealing to numerical instances. Also, in pre-modern Chinese mathematical documents before Jiao Xun, the dimensions of geometrical diagrams are usually drawn according to the numerical values in the problem to which the diagrams are attached, and diagrams are in fact seldom used to discuss purely arithmetical statements. However, Jiao Xun's case is totally different. Geometrical diagrams are used in discussing the four basic operations, and the dimensions are not assigned to any specific length, for the statements themselves do not have numerical values. Geometrical diagrams, it seems, are used to enhance the reader's intuitive understanding, which is an educational goal, because he wanted the readers to "see" the truth of a general statement without appealing to numerical calculations. In sum, as Li Huang's and Jiao Xun's works show, mid-Qing scholars had turned to a more generalized style of argumentation.

⁵⁵ Guo ed., *Zhongguo kexue jishu dianji tonghui*, Vol. 4, 1296: “以甲加乙，以乙加甲，則差平；以甲加甲，以乙加乙，則差倍。”

⁵⁶ Ibid.: “甲本盈，以乙消之。乙本朒，以甲補之。故有差而無差。... 甲盈又益以甲，乙朒止益以乙。有兩甲乙，即有兩甲乙之差，故倍之也。”

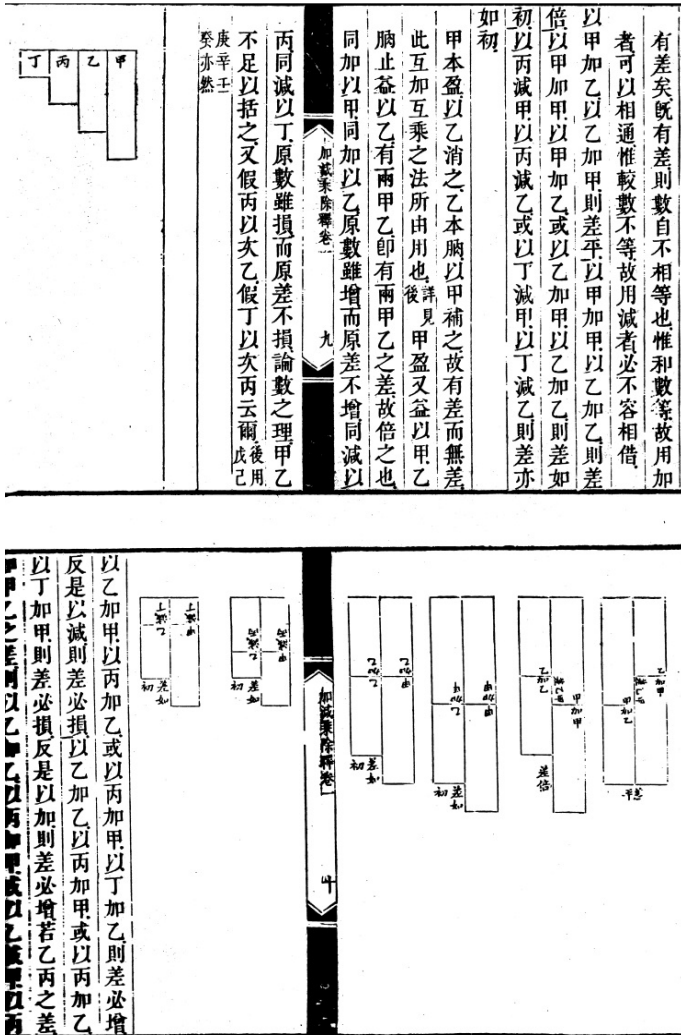


Figure 5. Jiao Xun's arguments and diagrams. Guo ed., *Zhongguo kexue jishu dianji tonghui: shuxue pian*, Vol. 4, 1296.

Concluding Remarks

With this evidence in hand, we can try to provide partial answers to the questions raised at the beginning of this article, especially about the influences of the two versions of the *Elements* and the meaning of “calculation principles.” As stated earlier,

the two Chinese versions of the *Elements* present very different approaches to the study of Euclidean geometry. Ricci and Xu's translation in 1607 gives an axiomatic-deductive system, while the *Shuli jingyun* version uses various ways, including intuitive, numerical, and even hands-on approaches, to argue for its propositions. However, at least until the late eighteenth century, the two texts did not seem to have mutually exclusive or contradictory influences. Mathematicians in the late Ming to early Qing period, and also the Emperor Kangxi himself, saw Euclidean geometry as a tool for real-world applications. Although a European flavor can be observed in geometrical diagrams, such as the labelled vertices, the organization of mathematical texts was still mainly problem-oriented. None of the texts written by Chinese scholars, beside Ricci and Xu's translation itself, was organized as an axiomatic-deductive system. Numerical examples and the solution of problems remain important elements in the works of Chinese scholars. The compilation of the *Shuli jingyun* did prompt Chinese mathematicians to reflect more upon their argumentative methods, and later scholars began to write texts whose arguments are more abstract than numerical. Although the focus was never on the logical structure of geometry, some scholars, such as Li Huang and Jiao Xun, did use a more abstract style to explain the principles of calculations, before using them to solve problems numerically.

We believe that in the previous paragraph we have described the combination of the influences of the two versions of the *Elements*. Chinese scholars did not fully embrace the axiomatic-deductive system, but with the desire and efforts of *huitong*, they gradually integrated argumentative styles from different traditions, and turned from numerical to general descriptions, though not without the help of intuition. One might argue that the pursuit of "generality" is also a trait of Chinese mathematics, as seen in Liu Hui's commentary for the *Jiuzhang suanshu*.⁵⁷ Again, while this paper does not object to this claim, this reason alone cannot explain the gradual change toward abstract arguments from the late Ming to the mid-Qing period. It is because in the Ming Dynasty when much of ancient Chinese mathematics, including most of the contents and commentaries of the *Jiuzhang suanshu*, was lost, Ming mathematicians before Xu Guangqi did not show a trend towards generality, and that trend started after the translation of Euclid's *Elements*, and was enhanced by the *Shuli jingyun*. And in the whole process, the Chinese efforts of *huitong* played an important role, as shown in this paper.

Coming back to the main question of this article, the meaning of *suanli*, or calculation principles, we can see that the two characters *suan* 算 (calculation) and *li* 理 (principles) both play very important roles. The mathematical structure of the axiomatic-deductive system was certainly not what they meant by *suanli*, even though very high praise was given to Ricci and Xu's translations. In the late Ming to early

⁵⁷ Karine Chemla, "Generality above Abstraction: The General Expressed in Terms of the Paradigmatic in Mathematics in Ancient China," *Science in Context* 16:3 (2003), 413-458.

Qing period, they used numerical instances, sometimes with exact diagrams, to help calculate the solutions to problems, and with that process they tried to convince the reader that the propositions in question were true. The “principles” were actually hidden in the “calculations.” In the early to mid-Qing period, they used more abstract styles to argue for the propositions, sometimes with the help of diagrams for readers to “see” the truth, and then they used the principles to solve ancient problems. So they argued for general “principles” in order to successfully solve and understand the “calculation” in classical problems. We believe that with the examples we have shown, we can say that what 17th and 18th century Chinese mathematicians meant by *suanli* was never the kind of pure deduction in the Euclidean manner, but *a combination of induction and deduction*, with the help of intuition, for the purpose of problem solving. And this kind of argumentative style was the product of the efforts of *huitong*. In a period of time when East Asian mathematicians tried to integrate European traditions into their own, there are still many aspects that can be studied, and hopefully even more interesting results can be found by scholars in the future.

Abstract

This article discusses the influence that two versions of the *Elements* had in Ming and Qing China as well as Chinese scholars' efforts to "integrate" (*huitong* 會通) Western and Chinese mathematics into a unified system in terms of argumentative styles in mathematics. Although much high praise was given to the axiomatic-deductive system of Euclid's *Elements*, numerical arguments and problem-solving remain important traits in the works of Chinese scholars. The compilation of the *Shuli jingyun* 數理精蘊 and the inclusion of another version of the *Elements* in it again prompted East Asian mathematicians to reflect more upon their methods of argumentation, and later scholars began to write texts whose arguments are more abstract than numerical. This paper presents examples taken from the works of several representative scholars from the late Ming to the mid-Qing periods to argue that their efforts of *huitong* produced argumentative styles that are a combination of both Chinese and Western approaches, and that there was a trend of moving from concrete calculations to general arguments in mathematical texts throughout the course of history. Finally, the authors conclude that what 17th and 18th century Chinese mathematicians meant by "calculation principles" (*suanli* 算理) was never the kind of pure deduction in the Euclidean manner, but a combination of induction and deduction, with the help of intuition, for the purpose of problem-solving.

Keywords: *suanli* (calculation principles), Chinese mathematics, late Ming to mid-Qing period, Euclid's *Elements*, *Shuli jingyun*