

The Origins and Heuristic Value of James Clerk Maxwell's Vortex Model of Electromagnetic Ether

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제임스 클러 맥스웰의 전자기에터 소용돌이 모형의 기원과 발견적 가치

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요 약

맥스웰의 두번째 전자기학 논문 「물리력선에 관하여」에 나타나는 유명한 소용돌이 에터 모델에 대하여 간략하게 고찰하고 그 모델의 발견법적 가치를 둘러싼 논쟁에 대하여 검토한다.

James Clerk Maxwell's second paper on electricity and magnetism, *On Physical Lines of Force* (1861-62), is one of the most problematic and interesting documents in history of science. It was in this paper that the most important innovations in Maxwell's study of electricity and magnetism—the concept of displacement current and the identification of light with transverse waves in electromagnetic medium—were first introduced. It was also distinguished by an extensive use of the famous vortex model of electromagnetic ether in almost all details of the arguments. To trace the course through which Maxwell was led to the innovations has presented a difficult problem of interpretation, especially with respect to the heuristic value of the specific mechanical model employed. In this short paper I have tried to understand the elusive interaction between the development of Maxwell's ideas and the molecular vortex model of electromagnetic ether in the context of *On Physical Lines* by investigating major historical literature touching the subject.¹⁾

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1) Joan Bromberg, "Maxwell's Displacement Current and His Theory of Light," *Arch. Hist. Exact Sci.* 4(1968) : 218-234 ; Nancy Nersessian, *Faraday to Einstein: Constructing Meaning in Scientific Theories*

Every Maxwell scholar agrees on the point that Maxwell's study of electricity and magnetism was profoundly influenced from the outset by Faraday and Thomson. Faraday's long research career in electricity and magnetism can be summarized in a word as a struggle to formulate electrical and magnetic phenomena by the patterns of vectorial forces propagating through the space, as opposed to the prevailing action-at-a-distance formulation based upon the notion of electrical and magnetic fluids. These spatial patterns were graphically delineated by "lines of force", whose direction represented the directions of the forces, while their spacing indicated the magnitudes of the forces. Faraday's rudimentary calculus of lines of force enabled him to deal with known electrical and magnetic phenomena effectively and proved immensely fruitful in suggesting new experiments that led to the discovery of novel phenomena.²⁾

It was Thomson who first identified Faraday's lines of force with mechanical conditions in a material medium, thus providing the background for a unified ether theory of electromagnetic and optical phenomena. Thomson's study of Fourier's theory of heat conduction in the early 1840s led him to recognize its mathematical similarity to the conventional formulation of electrostatics by Coulomb and Poisson. Also, by 1845, he had succeeded in showing that action-at-a-distance electrostatics and Faraday's calculus of electrical lines of force were mathematically equivalent, yielding in turn the mathematical analogy between Fourier's heat flow theory and Faraday's lines of force electrostatics. If one took the mathematical similarity to be suggestive of a physical similarity, the implication would be that, just as heat flow depends on a medium, so do Faraday's lines of force. Following this line of reasoning of physical analogy, Thomson argued that electrical forces and magnetic forces "may be discovered to be produced entirely by the action of contiguous particles of some intervening medium, and we have an analogy for this in the case of heat, where certain effects which follow the same laws are undoubtedly propagated from particle to particle."³⁾

As for the nature of the medium, it was also Thomson who introduced the concept of vortex motion. Obviously, the chief inspiration came from Faraday's discovery of magneto-optic rotation and Rankine's theory of molecular vortices. As a matter of fact, Faraday had already speculated on a possible relationship between the lines of force and light. He thought that "radiant phaenomena" might correspond to transverse undulations of just the lines of force rather than of the independent luminiferous ether, and his discovery of magneto-optic effect confirmed the connection between light and magnetism. Thomson, however, argued that Faraday rotation could be explained only on the assumption that the magnetic lines of force corresponded to the axes of rotation of the material medium through which the light propagated. In a letter dated 1847 to Faraday, Thomson stated that "if a physical theory of the propagation of electric and magnetic forces could be discovered, it would also, when taken in connection

(Dordrecht: Martinus Nijhoff, 1984) ; Daniel Siegel, "The Origin of the Displacement Current," *Hist. Stud. Phy. Sci.* 17 (1986) : 99-146 ; Alan Chalmers, "The Heuristic Role of Maxwell's Mechanical Model of Electromagnetic Phenomena," *Stud. Hist. Phil. Sci.* 17 (1987) : 415-427.

2) Daniel Siegel, "Thomson, Maxwell, and the Universal Ether in Victorian Physics," in G.N. Cantor and M.J.S. Hodge, eds., *Conceptions of Ether: Studies in the History of Ether Theories* (New York, 1981) : 239-268, p. 240.

3) Ibid., pp. 241-2.

with the undulatory theory of light, in all probability explain the effect of magnetism on polarized light."⁴⁾

Maxwell was deeply impressed by the power of Thomson's method of physical analogy. The method involves using a resemblance in form between the laws of different classes of phenomena as a means of obtaining physical ideas without a physical theory. Maxwell saw in this method a *via media* between the sterility of a pure mathematical analysis and the excesses of speculative physical hypotheses.⁵⁾ In investigating a new and underdeveloped class of phenomena, the physical analogy drawn from a well understood phenomena provides not only a mathematical formalism, but also a concrete, visual image to apply to the new phenomena. Maxwell, in 1855, wrote to Thomson about the method of physical analogy: "Have you patented that notion with all its implications? For I intend to borrow it for a season." The result of Maxwell's application of it to electrical and magnetic phenomena was *On Faraday's Lines of Force* (1855-56), where the lines of force were compared to the flow of an incompressible fluid through a resistive medium. After picking up Thomson's idea of the real rotation in the medium of magnetic action, Maxwell developed the famous mechanical analogy in *On Physical Lines*.

On Physical Lines appeared in 4 parts. Part 1 dealt with magnetic phenomena, part 2 with electric currents and electromagnetic induction, and part 3 with electrostatics. For the mechanical model to be consistent with known properties of magnetic lines of force, namely longitudinal tension and lateral repulsion, Maxwell in part 1 assumed that the magnetic medium consisted of tiny fluid vortices. The axis of rotation of each vortex being in alignment with the local direction of the line of force and the angular velocity being proportional to the field intensity, the equatorial expansion of the vortices due to the centrifugal force yielded those properties. Evaluating the associated stress tensor in the medium quantitatively, Maxwell was able to account for all the known magnetic phenomena.

The electrical particles introduced in part 2 served the double role of constituting electricity and making the magnetic vortices in a region rotate in the same direction. The electrical particles formed monolayer between each magnetic vortex cell, acting as idle wheels in the machinery. In a region of homogeneous magnetic field, where adjacent vortices have equal angular velocities, the particles act as ordinary idle wheels, rotating but undergoing no spatial translation. In an inhomogeneous magnetic field, however, adjacent vortices have slightly different angular velocities, giving rise to a translational motion of the idle wheel particles. Maxwell derived on a purely kinematic basis the relation between flux density of the particles and the angular velocity of the vortex, showing that the relation is mathematically identical to that between electric current density and magnetic field intensity expressed by Ampère's law, $\mathbf{J} = (1/4\pi) \text{curl } \mathbf{H}$. This led Maxwell to conclude "it appears that, according to our hypothesis, an electric current is represented by the transference of the movable particles interposed between the neighboring vortices."⁶⁾ He then considered the effect on the idle wheel

4) Ibid., p. 242.

5) Nersessian, op. cit., p. 73.

6) Siegel (1981), pp. 248-9.

particles of the temporal changes in the angular velocities of the vortices. The relationship obtained from mechanical inference was similar to Faraday's law of electromagnetic induction in differential form, $\text{curl } \mathbf{E} = -d/dt (\mu \mathbf{H})$. Letting the magnetic permeability correspond to the mass density of the medium, he claimed that the electromotive force, or the electric field, was conceptualized in the model by the tangential forces exerted by the vortices on the idle wheel particles.⁷⁾ By this scheme, Maxwell was able to explain the interaction between electric current and magnetic field consistently. One point that should be noted here is that the implication that the propagation of magnetic and electric phenomena in the medium definitely need time was clearly seen by this stage.

It is agreed unanimously by Maxwell scholars that there is every reason to believe that Maxwell originally intended to finish the paper with part 2. His vortex-particle model of electromagnetic ether proved to be quite successful, and the closing remarks of part 2 obviously showed that they were to be a summary and conclusion to the whole paper. Maxwell, however, ultimately went beyond it and undertook the task to incorporate electrostatics in his model. The important new property endowed in part 3 to the material forming the medium was elasticity. Each vortex was now regarded as a rotating, approximately spherical blob of elastic solid. Then, Ampère's circuital law considered earlier should be reexamined accordingly, and it was done in Proposition 14, "to correct equations of electric currents for the effect due to the elasticity of the medium".⁸⁾

We have seen that electromotive force and electric displacement are connected by equation (105). Differentiating this equation with respect to t , we find

$$dR/dt = -4\pi E^2 (dh/dt), \quad (111)$$

showing that when the electromotive force varies, the electric displacement also varies. But a variation of displacement is equivalent to a current, and this current must be taken into account in equations (9) and added to r . The three equations then become

$$\begin{aligned} p &= (1/4\pi) \{d\gamma/dy - d\beta/dz - (1/E^2)(dP/dt)\} \\ q &= (1/4\pi) \{d\alpha/dz - d\gamma/dx - (1/E^2)(dQ/dt)\} \\ r &= (1/4\pi) \{d\beta/dx - d\alpha/dy - (1/E^2)(dR/dt)\} \end{aligned} \quad (112)$$

where p, q, r are the electric currents in the directions of x, y , and z ; α, β, γ are the components of magnetic intensity; and P, Q, R are the electromotive forces.

Now if e be the quantity of free electricity in unit of volume, then the equation of continuity will be

$$dp/dx + dq/dy + dr/dz + de/dt = 0. \quad (113)$$

Differentiating (112) with respect to x, y , and z respectively, and substituting, we find

$$de/dt = (1/4\pi E^2) (d/dt) (dP/dx + dQ/dy + dR/dz); \quad (114)$$

whence

$$e = (1/4\pi E^2) (dP/dx + dQ/dy + dR/dz). \quad (115)$$

7) Ibid., p. 250.

8) James Clerk Maxwell, *The Scientific Papers of James Clerk Maxwell* (New York: Dover, 1952), ed. by W.D. Niven, vol. 1, pp. 496-7. Equation (105) reads $R = -4\pi E^2 h$, where R is the z -component of the electromotive force, h is the z -component of the displacement, and E is a constant varying with the nature of the dielectric. Equations (9) are Ampère's circuital law, $(1, c, 1) = (1/4\pi) (\alpha, \beta, \gamma)$.

This proposition has been the focus of considerable historical analyses and debate. One question immediately arising is precisely what Maxwell meant by (p, q, r) in equation (112). Since he called it just "electric current" and used the same notation p, q, r as in equation (9), it is not obvious at first glance whether he referred to the total current or to the conduction current alone. Bromberg has argued that the meaning of (p, q, r) in Proposition 14 was indeed ambiguous, in equation (112) corresponding to total current and in equation (113) to conduction current.⁹⁾ Subsequent studies by historians of science, however, have yielded an agreement that it represented conduction current alone, for it was (p, q, r) that went into the continuity equation. That interpretation is entirely consistent when the fact that displacement was a negative multiple of electromotive force in *On Physical Lines* is firmly recognized. Siegel has quite legitimately pointed out that Maxwell's "displacement" was not the concept that is designated by the same name today, which is of the same direction with electric field, but rather corresponded to what we would call electric dipole moment per unit volume.¹⁰⁾ The proper interpretation of equation (112) is, then,

$$\text{conduction current} = \text{curl } H/4\pi + (d/dt) \text{ displacement or}$$

$$\text{conduction current} = \text{curl } H/4\pi - (\text{const.}) (d/dt) \text{ emf}$$

The last interpretation can be readily assimilated to the modern, modified form of Ampère's law. It is in this sense that equation (112) constitutes the introduction of displacement current and the generalization of Ampère's law.

The origins and purpose of the introduction of displacement current have been one of the central questions in Maxwell scholarship. The popular, textbook account goes that Maxwell, comparing Ampère's circuital law with the continuity equation for electric charge, noticed the inconsistency between them, and thus added the (dD/dt) term to the circuital law to harmonize them with each other. Another version of the account places greater emphasis on Maxwell's aesthetic desire to obtain symmetry between Ampère's law and Faraday's law of electromagnetic induction. However, Bromberg has shown convincingly that, at least in the context of *On Physical Lines*, there is no direct evidence of Maxwell's aesthetic desire as such and little evidence supporting the claim that harmonizing Ampère's law and the continuity equation was important to him.¹¹⁾ He did compare Ampère's law and the continuity equation in *On Faraday's Lines* and was well aware of the limitation of Ampère's law that it was applicable only to closed circuits.¹²⁾ But Bromberg emphasizes the fact that the ultimate result of the key proposition 14 was formula (115), and the only use of proposition 14 made of by Maxwell lay in the derivation of Coulomb's law from formula (115). Moreover, Coulomb's law, thus obtained and compared with the same law in electromagnetic units, led Maxwell to find the value of E for vacuum, which turned out to be very close to the known speed of light. He then calculated the speed of propagation of a transverse undulation in the medium, obtaining the result $V^2 = m/p$, where m was the shear modulus and p was the mass density of the medium. From purely

9) Bromberg, op. cit., pp. 220-22.

10) Siegel (1986), p. 119.

11) Bromberg, op. cit., pp. 223-24.

12) Siegel (1986), pp. 108-9.

mechanical considerations he could show that the speed of propagation V equaled E for vacuum. Maxwell exclaimed that "the velocity of transverse undulations in our hypothetical medium..... agrees so exactly with the velocity of light.....that we can scarcely avoid the inference that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena."¹³⁾ Thus I agree with Bromberg that Maxwell's immediate intent in introducing displacement current term was to make it possible to compute E .

Closely related to this point is the debate about the heuristic value of the vortex-particle model in the development of Maxwell's idea. While J.J. Thomson once remarked that Maxwell's method represented one of the most striking examples of successful use of a model, Pierre Duhem at the other extreme contended that Maxwell had arrived at the notion of displacement current and the electromagnetic theory of light through other routes and later fudged up the model to fit in the theory. More recently, a sharp disagreement between Chalmers and Nersessian has been expressed with respect to this subject.

Chalmers' position is, however, different from the extreme ones, such as that of Duhem. Nersessian and Chalmers agree on that Maxwell's model was of "provisional and temporary" character and was adopted as essentially a heuristic device. Chalmers also rejects Duhem's contention that Maxwell fudged up the entire model after the development of displacement current and electromagnetic theory of light, recognizing the independence of Parts 1, 2 and Part 3. Chalmers' central argument against the heuristic value of the specific model is that the notion of displacement and displacement current and the relation $R = -4\pi E^2 h$ originated not from the necessary mechanical property of the medium, but from purely electrical considerations.¹⁴⁾ He takes Maxwell's remark, "these relations are quite independent of any theory about the internal mechanism of dielectrics," as a conclusive evidence for their independence from the model. Although Maxwell said right below of it "we cannot help regarding the phenomena as those of an elastic body," Chalmers emphasizes that that mechanical conceptualization was after the derivation of the concept by electrical reasoning independent from the model.¹⁵⁾

However, although it is difficult to deny that the idea of displacement induced in dielectrics was originally electrical in character, that is hardly sufficient to play down the role of the vortex-particle model in *On Physical Lines*. As far as Parts 1 and 2 are concerned, the model was devised to account for the known phenomena and worked quite well. It seems reasonable to assume that Maxwell, after devising the model for Parts 1 and 2, somehow hit upon the idea that the model, with some modification, could also incorporate electrostatics and provide the desired connection between light and electromagnetic phenomena. Without the model, Maxwell this stage would not have been able to make use of any electrical considerations or whatever. It is very important and suggestive that he defined displacement to be in the opposite direction to electromotive force, which was conceptualized as the force exerted by neighboring vortices on electrical particles. This cannot be understood except by identifying the electromotive force with the restoring force in the elastic medium.¹⁶⁾ Whatever the immediate origin of the notion

13) Maxwell, *Scientific Papers*, vol. 1, p. 500.

14) Chalmers, op. cit., p. 420.

15) Ibid., pp. 422-423.

16) Nersessian, op. cit., p. 83; Siegel (1986), pp. 112-113.

of displacement might have been, it was through the physical analogy to the specific model that Maxwell made sense of it. As a matter of fact, he did not even expound in this paper the rather obvious implications of displacement current, such as the magnetic field due to the static electricity, for he was only interested in the method of physical analogy as a heuristic device to lead himself to electrostatics and to the connection to light. The model was an integral, indeed a central, element in the development of Maxwell's thinking.